EE334: Electronic Design Lab
Analog Compensator Design Assignment

Group 6
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1 Design Process

From the given problem statement, first we require the transfer function (abbr., tf) $\frac{B(s)}{N(s)} = H_1(s)$, say to attenuate by 20 dB at 100 Hz frequency. Let $G(s)$ be the open loop tf without the compensating controller, and let $C(s)$ be the cascaded compensator we add to meet the specifications. Having defined these, we have

$$H_1(s) = \frac{G(s)}{1 + C(s)G(s)}$$

Now, according to the given data, we obtain the value of $G(s)$ at $f = 100$ Hz as approximately 18.76 dB which is roughly 8.67. Since, we want $H_1(s)$ to attenuate by 20 dB (by a factor of 10) at 100 Hz, we have

$$\begin{aligned} H_1(s) &= \frac{G(s)}{1 + C(s)G(s)} = \frac{1}{10} \\ \therefore\ C(s)G(s) &= 10G(s) - 1 \\ \therefore\ C(s)G(s) &= (10 \times 8.67) - 1 \\ \therefore\ C(s)G(s) &= 85.7 \approx\ 38.66 \text{ dB} \end{aligned}$$

All the above calculations are at $s = 2\pi j \cdot (100)$.

Therefore, as the results of these calculations suggest, we want the magnitude plot of the final open loop tf ($H_{ol}(s) = C(s)G(s)$) to attain 38.66 dB at $f = 100$ Hz.

Now, let's say we add this excess gain (38.66 dB − 18.76 dB = 19.9 dB) via a simple proportional controller. The corresponding Bode plots are shown in Figure 1.

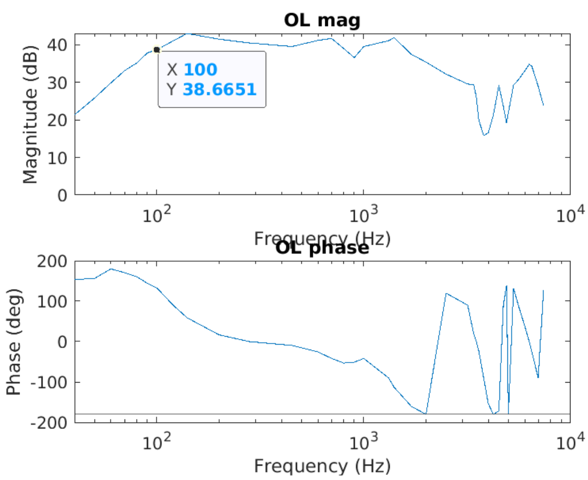

Figure 1: Bode plots of the plant after a gain of 19.9 dB is added

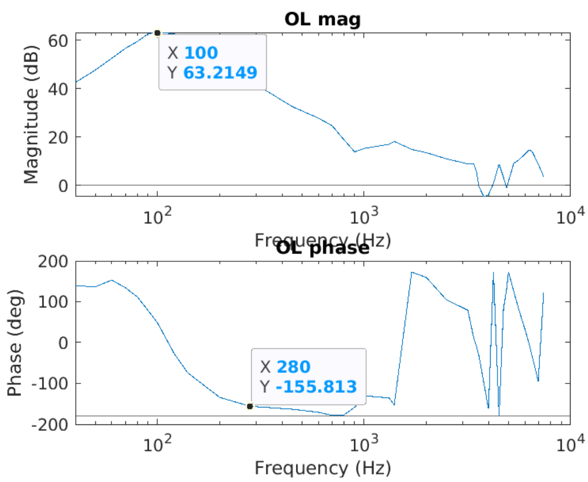

Figure 2: Open loop Bode plots after the introduction of the second-order lag compensator

Clearly, wherever the phase Bode plot crosses 180°, the magnitude Bode plot has value greater than 0 dB, which implies instability of the closed loop. Therefore, we need a mechanism through which we can have all gain margins (corresponding to multiple phase crossover frequencies) positive, and adequate phase margins (corresponding to the gain crossover frequencies).

Therefore, we clearly require a lag compensator so that we get a much faster decaying Bode magnitude plot which gives us appropriate gain and phase margins. We chose a **second-order lag compensator** as a starting point to meet the specifications. The motivation behind this choice is that a second-order lag compensator has a much steeper roll-off in the transition region as compared to a first-order lag compensator. Our requirement is that the Bode magnitude plot decays off quickly before the phase crosses 180°for the first time so that all gain margins are positive. Therefore, we select the following tf for the cascade compensator:

$$C_1(s) = \frac{s^2 + 2 \times (0.3) \times (2\pi \cdot (1010.15))s + (2\pi \cdot (1010.15))^2}{s^2 + 2 \times (0.3) \times (2\pi \cdot (101.015))s + (2\pi \cdot (101.015))^2}$$

The damping ratio is chosen to be 0.3 for both the underdamped systems and the natural frequency for the numerator system is 1010.15 Hz and that for the denominator system is 101.015 Hz. The damping ratio is chosen somewhat arbitrarily by hit-and-trial. However, the natural frequency is chosen so that the resonant peak introduced by the denominator second-order tf is around 100 Hz. The Bode plots shown in Figure 2 for the open loop were observed.

Now, the problem with the above Bode plot is that the gain crosses over much later than the phase and leads to instability of the closed loop. Also, we see that apart from the phase crossover points beyond 1 kHz, there is even a point before 1 kHz where there is a potential phase crossover. We want to design the system in a way such that the phase crossovers beyond 1 kHz remain somewhat as they are, and at the same time we want to do away with phase crossover frequencies less than 1 kHz. Therefore, we decided to introduce a phase lead through a **first-order lead compensator**, such that this point of concern in the phase plot is taken care of.

Using information from `https://www.sciencedirect.com/topics/engineering/lead-compensator`, we design the compensator. We introduce a phase lead of around 30°, which implies that the value of $a = 3$, according to the source. Also, the value of $w_m = 2\pi \cdot 250$ rad/s, which is the point of maxima of the compensator's phase plot. Therefore, we arrive at the following tf for the additional phase-lead compensator:

$$C_2(s) = \frac{1 + (3 \times 3.67 \times 10^{-4})s}{1 + (3.67 \times 10^{-4})s}$$

The Bode plots shown in Figure 3 for the modified open loop were observed.

We see that the lead compensator successfully introduces the required phase lead around the mentioned frequency (around 250 Hz), and also prevents the phase crossover just before 1 kHz. Now, we again observe that there is a possible phase crossover between 1 Hz and 100 Hz. So, we finally add a simple first-order lag compensator with corner frequencies $f_z = 70$ Hz and $f_p = 40$ Hz (arrived at after various tries) that introduces a phase lag roughly between 8 Hz and 350 Hz. Along with this, we also add a suitable gain so that we meet the 38.66 dB open loop magnitude specification at 100 Hz frequency.

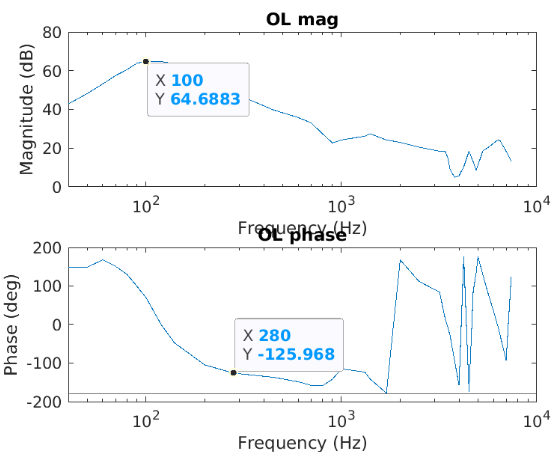

Figure 3: Open loop Bode plots after the introduction of the first-order lead compensator

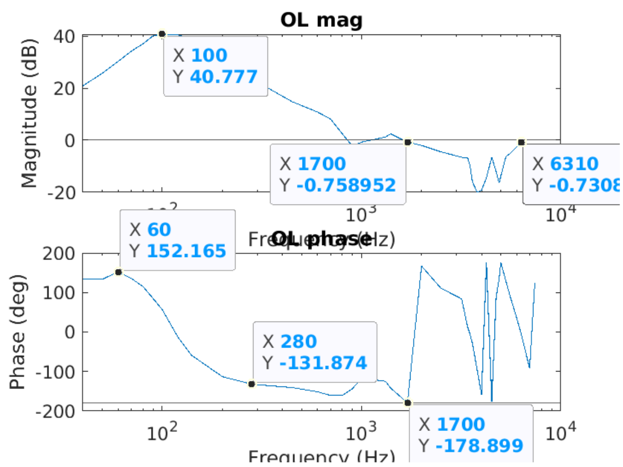

Figure 4: Final open loop Bode plots after the introduction of 1^{st}-order lag compensator

Therefore, we arrive at the following tf for the additional phase-lag compensator:

$$C_3(s) = \frac{1}{17.78} \times \left(\frac{s + 140\pi}{s + 80\pi} \right)$$

The final Bode plots for the open loop are attached in Figure 4.

For the final open loop (plant + compensator, where the compensator $C(s) = C_1(s)C_2(s)C_3(s)$), we make the following commments:

1. All the potential phase crossover points before 1 kHz have been eliminated
2. The required specification of at least 38.66 dB open loop magnitude at 100 Hz is met
3. At all the phase crossover frequencies (starting at around 1.7 kHz), the magnitude plot is below 0 dB

For the final system (refer Bode plots in Figures 5 and 6), we make the following observations:

1. The magnitude Bode plot of the closed-loop tf from the noise to output indeed falls to around -22 dB
2. The magnitude Bode plot of the closed-loop tf from the input to output is nearly constant at around 0 dB upto around 450 Hz, which comprises of normal human frequency bands

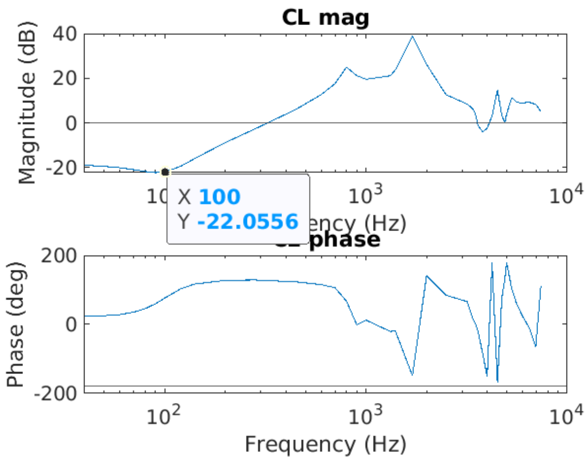

Figure 5: Bode plots of tf from noise to speaker

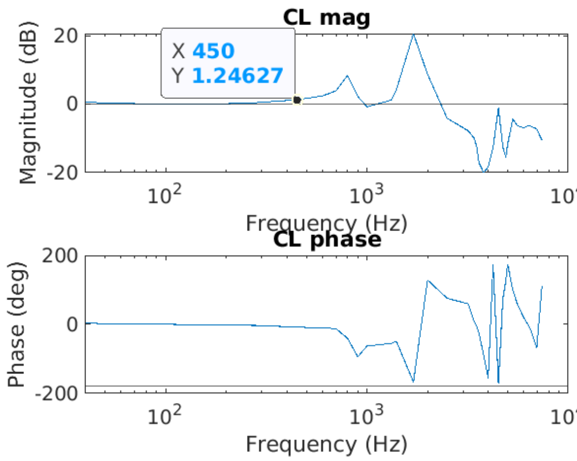

Figure 6: Bode plots of the final closed loop tf

The Bode plots obtained using MATLAB for the overall compensator ($C(s) = C_1(s)C_2(s)C_3(s)$) are shown in Figure 7.

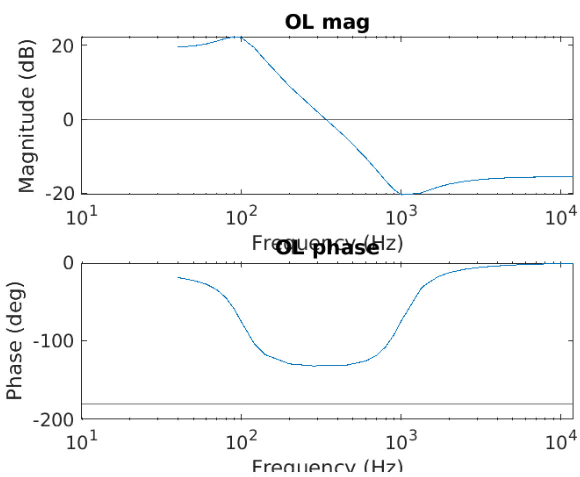

Figure 7: Bode plots of the overall compensator

The analog circuit (made up of a Tow Thomas bi-quad filter and inverting active first-order lag/lead networks) that simulates the compensator tf is shown below (Figure 8) along with the Bode plots from the circuit simulation (Figure 9, the dark solid line represents the magnitude plot and the faint dotted line represents the phase plot):

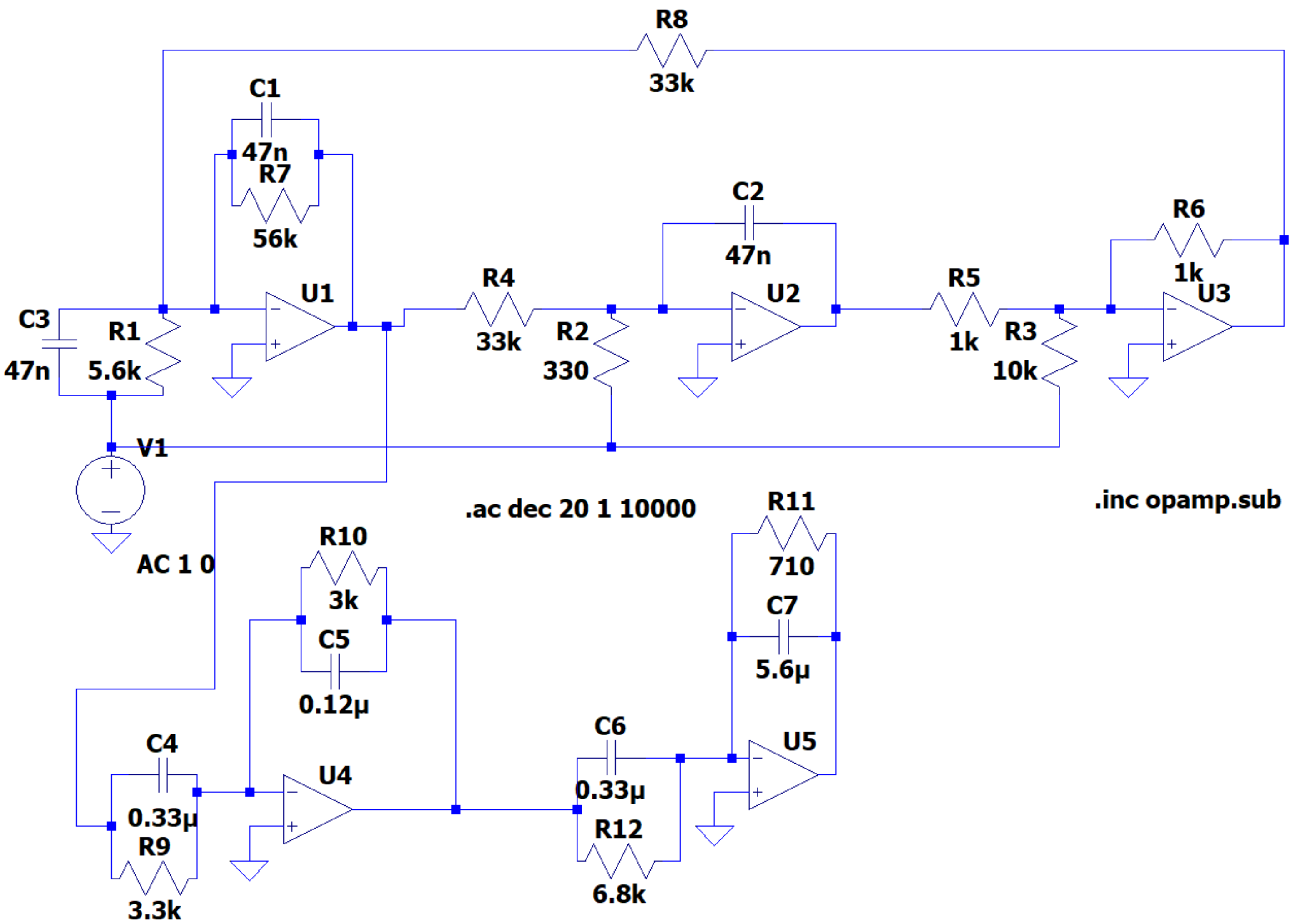

Figure 8: Analog circuit that simulates the designed compensator

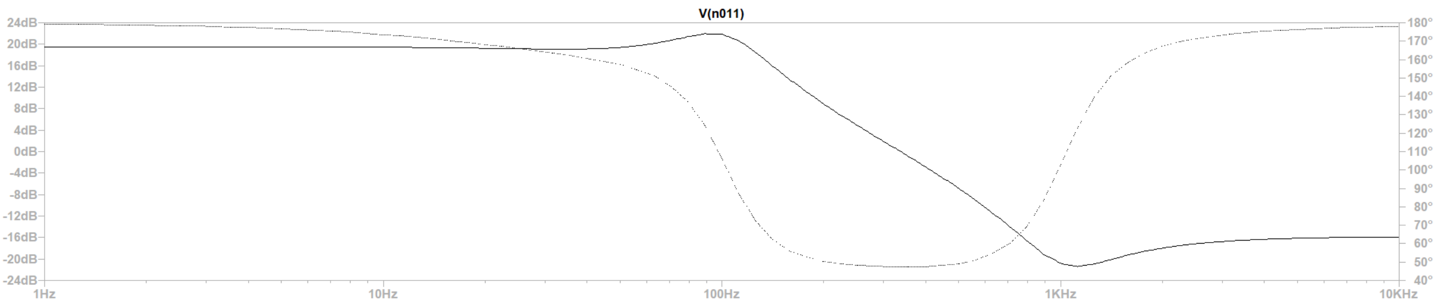

Figure 9: Bode plots obtained from analog circuit

2 Appendix

2.1 Code

The MATLAB code used for designing the system and making the Bode plots can be found at: `https://github.com/advaitkumar3107/EE-344/blob/main/compensator.m`